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The Polytope of Hereditary Information the Structure, Location, Signification

Gennadiy Vladimirovich Zhizhin*

Affiliation: Russian Academy of Natural Sciences, St Petersburg, Russia

***Corresponding author:** Gennadiy Vladimirovich Zhizhin, Russian Academy of Natural Sciences, St. Petersburg, Shotman Street, house 18, apartment 63, Russia 193232, Tel: +79932203662, Email: genzhizhin@gmail.com

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Abstract

The geometry of the neighborhood of the compound of two nucleic acid helices with nitrogen bases was investigated. It is proved that this neighborhood is a cross-polytope of dimension 13 (polytope of hereditary information), in the coordinate planes of which there are complementary hydrogen bonds of nitrogenous bases. The structure of this polytope is defined, its image is given. The total incident flows from the low-dimensional elements to the higher-dimensional elements and vice versa of the hereditary information polytope are calculated equal to each other. High values of these flows indicate a high intensity of information exchange in the polytope of hereditary information that ensures the transfer of this information.

Keywords: Hereditary information, Nucleic acids, Polytopes, DNA sequences, Carbon cycle.

Introduction

In 1953, James Watson and Francis Crick based on the analysis of diffractograms proposed a three-dimensional model of a DNA molecule consisting of two chains twisted into a spiral. At the same time, sugar molecules and phosphoric acid residues included in the DNA molecule were also considered three-dimensional. However, it was not indicated which three-dimensional figures correspond to sugar molecules and phosphoric acid residues. According to this model, flat nitrogenous bases, being in different chains, using two complementary hydrogen bonds connected two spirals into a single whole. But how exactly the flat nitrogenous bases are located in space remained unknown. And this is despite the fact that it is these relations that determine the most important issue of the transmission of hereditary information. Recently, it was proved that sugar molecules and phosphoric acid residues have the highest dimension. The phosphoric acid residue has a dimension of 4, and the sugar molecule has a dimension of 12. In this regard, the geometry of nucleic acids was considered taking into account the highest dimension of the components [1-6].

In addition, for the convenience of images, a simplified three-dimensional model of a sugar molecule was built on the basis of a full twelve-dimensional model of a sugar molecule. This model can be used in further constructions, not forgetting the dimension of its complete model. This model was used to construct images of single nucleic acid helices. When passing to the analysis of nucleic acids consisting of two linked helices, it is necessary to take into account the emerging anti-parallelism of geometric elements in linked spirals. In this article, continuing the research begun in the works, it will be shown that sugar molecules located in linked nucleic acid helices form a polytope of dimension 13 of the type of cross-polytope in which there are exactly 12 coordinate planes. In these coordinate planes outlined by rectangles with antiparallel edges, exactly 12 compounds of nitrogen bases currently known can be located. The image of the polytope and its coordinate planes is obtained, its structure is determined. This polytope can be called the polytope of hereditary information, since it transmits hereditary information from one spiral to another using a sequential alternation of nitrogenous bases. It has been shown that the polytopic of hereditary information is characterized by a powerful stream of incidents between geometric elements of different dimensions, providing an extremely intense exchange of information between the components of nucleic acids. The evidence of the existence of such an exchange of information can be found in the recently discovered inheritance of changes not related to the modification of sequences in DNA, i.e. with methylation-the binding of a methyl group CH_3 to the nitrogenous base of DNA [7,10-12].

Spatial modes of sugar molecule and nucleic acid helices

From a sugar molecule with five carbon atoms in the Fischer form, we can move on to representing the molecule as a closed chain. Having chosen, for example, Form B from the two enantimorphic forms A and B, the closed chain of the sugar molecule has the form in **Figure 1** [2,5-7,13].

To obtain a spatial figure of a sugar molecule (B-ribose), considering the atoms and functional groups shown in the projection onto the plane in Figure 1 as the vertices of the corresponding spatial figure, it is necessary to introduce additional edges in addition to the edges shown in Figure 1. These edges will have only spatial significance, unlike the edges in Figure 1, which also depict chemical covalent bonds. To distinguish between edges, we will depict edges corresponding to chemical covalent bonds with thick solid black lines. Edges that have only spatial significance in the images of the figures will be indicated by thin dashed black lines [7].

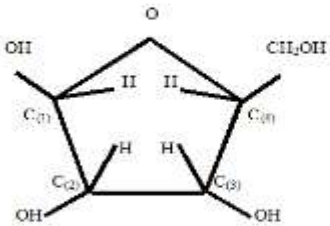

Figure 1: Closed chain sugar molecule (form B).

Since the number of atoms and functional groups is 13, the closest convex closed figure that can be obtained from Figure 1 is a simplex of dimension 12. The dimension of the simplex is n-1, where n is the number of vertices. Moreover, each vertex is connected to all other vertices by edges, i.e. 12 edges emanate from each vertex. A spatial image of the B-ribose molecule is given in the article. If, as a simplification, we leave, in addition to the edges corresponding to chemical bonds, only the edges of the external contour in the projection of the spatial figure onto the plane, we get an image of the B-ribose molecule, as shown in **Figure 2** [2].

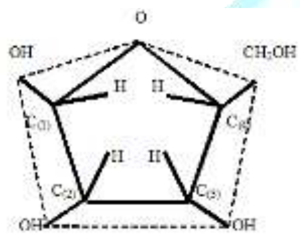

Figure 2: Schematic representation of a sugar molecule on a plane.

Since the lengths of the carbon-carbon and carbon-oxygen chemical bonds are close, we can approximately consider them equal to a=0.15 nm. In addition, it can be assumed that the edges of chemical bonds emanating from four carbon atoms in the carbon cycle are located symmetrically with respect to the plane of the carbon cycle. Then, as mathematically proved, chemical bonds emanating from four carbon atoms of the carbon cycle are perpendicular to the plane of the cycle. This leads to a simplified spatial image of the B-ribose molecule in **Figure 3**.

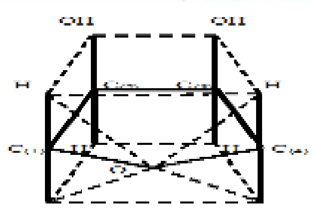

Figure 3: A simplified spatial image of a B-ribose molecule.

In Figure 3, of the additional edges needed to construct a convex closed spatial figure and to fulfill the Euler-Poincare equation, only the edges are left to give the figure a closed form. In this image, the B-ribose molecule is three-dimensional. But we must remember that the number of edges for fulfilling the Euler-Poincare equation should be much larger-12 edges from each vertex. The remainder of phosphoric acid H_3PO_4 is geometrically a tetrahedron with a center. Its dimension is 4. Having taken such images, the nucleic acid chain is as shown in **Figure 4** [7,14].

In Figure 4, in the sugar molecules, the functional group CH_2OH is replaced by some nitrogenous bases as a result of the isolation of water molecules.

Polytopes with antiparallel edges

In single-stranded and double-stranded nucleic acids (RNA, DNA), the constituents of acids (residues of phosphoric acid and sugar molecules) interact with each other. Phosphoric acid residues are connected by divalent metal ions, mainly magnesium ions, due to the interaction of negative charges of phosphoric acid residues with positive charges ions. This interaction is essential for the stability of nucleic acid structures, especially in the ribosomes. Sugar molecules interact with each other due to the hydrogen bond between the nitrogenous bases attached to the sugar molecules. Being geometric forms, the constituents of nucleic acids interact with each other to form new geometric forms-new polytopes. However, it is not known how flat nitrogenous bases are oriented in space, whether their orientation depends on the type of nitrogenous base. Currently there is no information on this. There is also no information on how exactly the metal ions are located, connecting the phosphoric acid residues. It should be remembered here that the adopted three-dimensional model of the components and the nucleic acid molecule itself is only a model for visual perception. As it was shown earlier, the phosphoric acid residue is a polytope of dimension 4, and the sugar molecule has a dimension of 12 [1,15-18].

Figure 4: Simplified nucleic acid chain.

The movement of triangles along a helix leads to the formation of polytopes with antiparallel edges. Consider an arbitrary triangle ABC on the plane. Choose some point O/ on the plane outside the triangle to his left. Let this point be the base of the axis of the helix passing through the triangle. Rotate the ABC triangle 180 degrees to the right, moving it up the helix, parallel to the initial plane. In the projection on the plane, both triangles ABC and the displaced triangle $A^{/}B^{/}C^{/}$ will be located as shown in **Figure 5**.

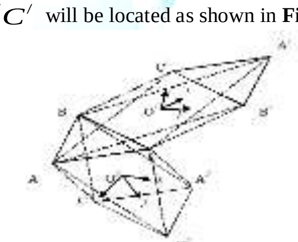

Figure 5: Polytopes of dimension 3 with anti-parallel edges.

It is easy to see that the edges of the triangle ABC and $A^{/}B^{/}C^{/}$ are antiparallel. It can now connect in space the vertices of the triangle ABC with the vertices of the triangle $A^{/}B^{/}C^{/}$ so that there are no connections of the vertices with the same letters. In a projection on the plane the connection are represented by dotted segments. It can be seen that the connecting segments also break up into pairs of anti-parallel segments. Let us now verify that the image ABCA/B/C/ in Figure 5, along with the dotted segments, is a projection of a three-dimensional convex polytope. One use the Euler-Poincaré equation for this aim f_k is the number of elements of dimension k in polytope of dimension n [14],

$$\sum_{k=0}^{n}(-1)^k f_k(n) = (-1)^k + 1, \qquad (1)$$

The shape ABCA/B/C/ in Figure 5 has 6 vertices, 12 edges, 8 triangular faces (rectangles are not faces by construction, since connecting, for example, vertex A with vertices B', C' it turns out to be exactly the triangle $AB'C'$). Substituting these values of elements of different dimensions into equation (1), can to find 6-12+8=2, i.e. the Euler-Poincaré equation holds in this case for n=3. This proves that the resulting figure is a convex polytope of dimension 3 (if the figure were not convex, the Euler-Poincaré equation would be violated). The point O/ in Figure 5 coincides with the center of the three-dimensional figure ABCA/B/C/ as diagonal figures pass through it and point O' located on the axis of the helix. Point O' can be considered as the origin of three-dimensional space. Coordinate axes x, y, z in this direction emanate from directions AA', BB', CC' respectively. Three pairs of these axes define the coordinate planes of the space of this shape. Choose some point O// on the plane outside the triangle below it. Let this point be the base of the axis of the helix passing through the triangle. Rotate the ABC triangle 180 degrees to the left, moving it up the helix, parallel to the initial plane. In the projection on the plane, both triangles ABC and the displaced triangle $A''B''C''$ will be located as shown in Figure 5.

It is easy to see that the edges of the triangle ABC and $A''B''C''$ are antiparallel. It can now connect in space the vertices of the triangle ABC with the vertices of the triangle $A''B''C''$ so that there are no connections of the vertices with the same letters. In a projection on the plane the connection are represented by dotted segments. It can be seen that the connecting segments also break up into pairs of anti-parallel segments. Let us now verify that the image ABCA/B/C/ in Figure 5, along with the dotted segments, is a projection of a three-dimensional convex polytope. Obviously, a shape ABCA//B//C// has as many vertices, edges, and flat faces as a shape ABCA/B/C/. Therefore, it satisfies the Euler-Poincaré equation (1) with dimensionality n=3, i.e. it is a convex three-dimensional polyhedron. The point O// in Figure 5 coincides with the center of the three-dimensional figure ABCA//B//C// as diagonal figures pass through it and point O'' located on the axis of the helix. Point O'' can be considered as the origin of three-dimensional space. Coordinate axes x, y, z in this direction emanate from directions AA'', BB'', CC'' respectively. The order of the coordinate axes x, y, z in the figures ABCA/B/C/ and ABCA//B//C//, as can be seen from Figure 5, coincide.

This suggests that both figures ABCA/B/C/ and ABCA//B//C//are topologically one and same figure-the wrong octahedron. Interestingly, to transform an arbitrary tetrahedron $ABCD$ into a tetrahedron $A'B'C'D'$ with anti-parallel edges, it is not enough to rotate it along a helix by 180 degrees. To do this, you must turn the helix together with the tetrahedron and move the tetrahedron along the helix in the opposite direction, also rotating it 180 degrees. In the initial state, the tetrahedron on the initial helix and the tetrahedron on the reversed helix, after its rotation by 180 degrees, will have anti-parallel edges. Both tetrahedrons can ABCD and $A'B'C'D'$ be present by the **Figure 6** for rotation to the right. On Figure 6 the point O/ is the point of rotation.

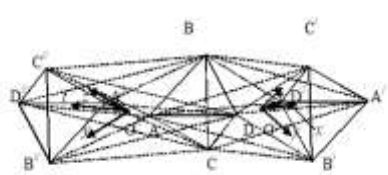

Figure 6: Polytope of dimension 4 with anti-parallel edges.

Now connect the vertices of the tetrahedrons so that the connecting edges (dotted segments) do not have the same letters. The resulting figure (along with dotted edges) has 8 vertices (f_0=8), 24 edges $(f_1 = 24)$, 24 triangular faces ($(f_2 = 24)$, and 8 tetrahedrons (f_3=8). Substituting these values into equation (1), can to find 8-24+24-8=0. Consequently, the Euler-Poincaré equation is satisfied in this case for n=4. Thus, the polytope ABCD $A'B'C'D'$ in Figure 6 has dimension 4. It is easy to see that this is 4-cross-polytope. The point O/ in Figure 6 coincides with the center of the fourth-dimensional figure ABCD $A'B'C'D'$ as diagonal figures pass through it and point O/ located on the axis of the helix. Point O' can be considered as the origin of fourth-dimensional space. Coordinate axes x, y, z, t in this direction emanate from directions AA', BB', CC', DD' respectively. Six pairs of these axes define the coordinate planes of the space of this shape [6,19,20].

Choose some point O// on the plane outside the tetrahedron ABCD below it. Let this point be the base of the axis of the helix passing through the tetrahedron. Rotate the ABCD tetrahedron 180 degrees to the left, moving it up the helix, parallel to the initial plane. In the projection on the plane, both tetrahedrons ABCD and the displaced tetrahedron $A''B''C''D''$ will be located as shown in Figure 6. It is easy to see that the edges of the tetrahedrons ABCD and $A''B''C''D''$ are antiparallel. It can now connect in space the vertices of the tetrahedron ABCD with the vertices of the tetrahedron $A''B''C''D''$ so that there are no connections of the vertices with the same letters. In a projection on the plane the connection is represented by dotted segments. It can be seen that the connecting segments also break up into pairs of anti-parallel segments. Let us now verify that the image ABCD $A''B''C''D''$ in Figure 6, along with the dotted segments, is a projection of a fourth-dimensional convex polytope. Obviously, a shape ABCD $A''B''C''D''$ has as many vertices, edges, and flat faces as a shape ABCD $A'B'C'D'$. Therefore, it satisfies the Euler-Poincaré equation (1) with dimensionality n=4. Thus, the polytope ABCD in Figure 6 has dimension 4. It is easy to see that this is 4-cross-polytope [6].

The point O// in Figure 6 coincides with the center of the 4-cross-polytope as diagonal figures pass through it and point $0''$ located on the axis of the helix. Point O'' can be considered as the origin of fourth-dimensional space. Coordinate axes x, y, z, t in this direction emanate from directions AA'', BB'', CC'', DD'' respectively. From Figure 6 it follows that the sequence of alternation of coordinate axes x, y, z, t in a 4-cross-polytope ABCD $A'B'C'D'$ differs from the sequence of alternation of coordinate axes x, y, z, t in a 4-cross-polytope ABCD $A''B''C''D''$. Thus, a surprising fact emerged: the figures ABCD $A'B'C'D'$ and ABCD $A''B''C''D''$, being 4-cross-polytopes, are topologically different from each other. It is impossible to move from one of them to another by continuous transformation, since they have a different order of alternation of vertices.

The polytope of hereditary information

Let us consider in detail the formation of a polytope of two sugar molecules with anti-parallel edges. Here, as in the case of the tetrahedron, to form a polytope with antiparallel edges from two sugar molecules, you must have one sugar molecule on one helix to turn this helix together with the sugar molecule and move the sugar molecule along this reversed helix in opposite direction to the original helix direction. When the sugar molecule rotates 180 degrees to the right while moving, then the original sugar molecule and the sugar molecule on the reversed helix are two polytopes with anti-parallel edges. Both of these sugar molecules in a simplified form are represented in **Figure 7.**

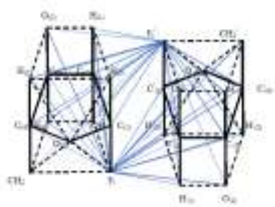

Figure 7: The two sugar molecules in a simplified form with anti-parallel edges.

When the five-carbon sugar molecule is rotated to the left by 180 degrees, the nitrogenous bases are on opposite sides of both molecules, so that for their connection it is necessary to cross the entire set of atoms of two molecules. This is unrealistic, therefore this option is not considered. When the sugar molecule is rotated 180 degrees to the left, it is possible to connect the sugar molecules through nitrogenous bases only between two different chains of nucleic acids or in the case of turning the chain itself in the opposite direction.

In full, the sugar molecules have a dimension of 12; in the corresponding polytope each vertex must have an edge connection with all the other vertices. Knowledge of this now one need for the formation of a polytope of dimension 13, it is necessary to connect each vertex of one polytope with the vertices of another polytope so that there are no vertex connections with the same letters. All connecting edges break into pairs of antiparallel edges. At the same time, a set of coordinate a two-dimensional plane emanates from the center of the formed polytope as from the origin of coordinates. Their number is equal to the number of combinations from 13 to 2, i.e. 48 coordinate planes. In order to clarify the possible geometrical circumstances of the connection of helices in double-helix nucleic acid molecules with nitrogenous bases, we are primarily interested in the coordinate planes containing these nitrogenous bases $F_i, F_i^{/}$.

There are exactly 12 such coordinate planes in the obtained polytope of dimension 13. They are depicted in Figure 4 by blue solid lines and are indicated below by the vertices of the polytopes contained in them

$$F_i'H_{(1)}\ \ F_i'O_{(2)}\ \ F_i'H_{(2)}\ \ F_i'C_{(3)}\ \ F_i'C_{(2)}\ \ F_i'H_{(3)}$$
$$H'_{(1)}F_i\ \ O'_{(2)}F_i\ \ H'_{(2)}F_i\ \ C'_{(3)}F_i\ \ C'_{(2)}F_i\ \ H'_{(3)}F_i$$
$$F_i'C_{(4)}\ \ F_i'H_{(4)}\ \ F_i'OH\ \ F_i'C_{(1)}\ \ F_i'O_{(1)}$$
$$C'_{(4)}F_i\ \ H'_{(4)}F_i\ \ (OH)'F_i\ \ C'_{(1)}F_i\ \ O'_{(1)}F_i$$

Other edges of the polytope of dimension 13 are not shown in Figure 7, so as not to ignite the figure. In the center of each parallelogram, indicated by its four vertices, is the origin of coordinates and the corresponding pair of coordinate axes (they are not shown). To identify the different hydrogen and oxygen atoms at vertices of polytope, they indicated by numbers in brackets at the lower indices. It is surprising that the number of coordinate planes containing vertices is exactly equal to the number of possible compounds of nitrogenous bases 12 [9].

$$A:U, U:A, G:C, C:G, G:U, G:A, U:U, U:C, A\cdot A, A\cdot C, U:G, A:G.$$

Since each coordinate plane designated by the vertices of the parallelograms has a specific atomic environment, it can be assumed that each of the 12 possible compounds of nitrogenous bases is located on one particular coordinate plane out of 12 possible. This solves the question of the possible orientation of the bond of nitrogenous flat bases in nucleic acids using ideas about the high dimensionality of the constituent nucleic acids. It is also surprising that in order to create 13-cross-polytopes, providing the connection with nitrogenous bases $F_i, F_i^{/}$, nature specially created double-stranded nucleic acids with oppositely directed spirals. This is realized in DNA and RNA when creating regions with inverted helices.

In a variety of nucleic acid molecules, the issue of chain interaction is important. In ribosomes, RNA interacts with each other due to bivalent metal ions (mainly magnesium ions). Positively charged magnesium ions attract negative charges of phosphoric acid residues, ensuring the stability of the ribosomes. In double-stranded nucleic acids, the helices are connected to each other by means of nitrogenous bases complementarily interacting with each other by a hydrogen bond. However, the magnesium ions and nitrogen bases in nucleic acids could not be specifically located. It has been established that magnesium ions and flat nitrogenous bases can be located inside special polytopes of higher dimension. Here knowledge is needed of the higher dimension of phosphoric acid residues and sugar molecules. Such polytopes are polytopes with anti-parallel edges, i.e. cross-polytopes of higher dimension. Binding agents are located on the free coordinate planes of these polytopes in the vicinity the center of the polytope.

In this case, the two-dimensional coordinate plane on the boundary of the polytope should contain the objects to be joined. In the case of magnesium ions, there are four specific coordinate planes inside the 5 cross-polytope, in which an ion can accommodate, combining negative charges. In the case of nitrogenous bases, the existence of 12 coordinate planes inside a cross-polytope of dimension 13, in which flat nitrogenous bases can be located, connecting the helix of nucleic acids, was discovered. Exactly as much as there are options for combining nitrogenous bases. It is given, that each coordinate plane of these 12 planes has a specific environment of atoms. It should be assumed that only one of the 12 possible compounds of nitrogenous bases is placed in each of these planes. It is surprising that the existence of higher-dimensional polytopes with anti-parallel edges is possible only in the case of the opposite direction of interacting helices, and this is exactly what nature provides in the double-helix DNA and in the RNA segments with self-inversion of the helix in the opposite direction.

To build the polytopic of hereditary information, we need to supplement Figure 6 with edges that create a closed and convex figure. In this case, it is not necessary to connect the vertices symmetrically relative to the center of Figure 6 with the edges. Each vertex will be connected by an edge to the other vertices. In this case, the polytope will be a cross-polytope and its dimension is equal to half the number of vertices. Indeed, according to, there is a relation between f_i the number of dimension elements i in the cross-polytope and the dimension d of the cross-polytope itself

$f_i(d) = 2^{1+i} C_d^{d-1-i}$. At the vertex i=0, therefore, according to (2) $f_0 = 2C_d^{d-1} = 2d, d = f_0/2 = 13.$ Thus, the polytope of the hereditary information has dimension 13. When portraying this polytop, we will use the technique that is used in portraying cross-polytops. We distribute all 26 vertices on the circle so that the vertices opposite in Figure 7 remain opposite and there is no edge between them **(Figure 8)** [6].

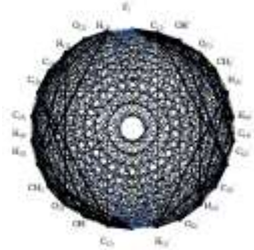

Figure 8: The polytope hereditary information.

The edges corresponding to the chemical bonds are indicated in this figure by thick black solid lines. Anti-parallel edges highlighting coordinate planes with vertices corresponding to nitrogenous bases are indicated by blue lines. The remaining edges are indicated by thin dash-dotted black lines. We emphasize that there is a chemical bond between the nitrogenous bases, but it is not covalent. Therefore, this connection is not indicated by an edge.

When the polytope of hereditary information moves along the common axis of the double helix of nucleic acids following the sequence of bases in DNA, the connection between the vertices $F_i^{/}, F_i^{//}$ is carried out by one of the twelve base pairs, which occupies one of the 12 coordinate planes of the polytope. By relation (2), one can determine the number of elements of different dimensions in the polytope of hereditary information.

The number of edges is $f_1(13) = 2^2 C_{13}^{11} = 312$,

The number of triangles is $f_2(13) = 2^3 C_{13}^{10} = 2288$,

The number of tetrahedrons is $f_3(13) = 2^4 C_{13}^{9} = 10560$,

The number of fourth-dimensional faces (simplexes) is $f_4(13) = 2^5 C_{13}^{8} = 41184$,

The number of fives-dimensional faces (simplexes) is $f_5(13) = 2^6 C_{13}^{7} = 109824$,

The number of six-dimensional faces (simplexes) is $f_6(13) = 2^7 C_{13}^{6} = 219648$,

The number of seven-dimensional faces (simplexes) is $f_7(13) = 2^8 C_{13}^{5} = 164736$,

The number of eight-dimensional faces (simplexes) is $f_8(13) = 2^9 C_{13}^{4} = 183040$,

The number of nine-dimensional faces (simplexes) is $f_9(13) = 2^{10} C_{13}^{3} = 292864$,

The number of ten-dimensional faces (simplexes) is $f_{10}(13) = 2^{11} C_{13}^{2} = 159744$,

The number of eleven-dimensional faces (simplexes) is $f_{11}(13) = 2^{12} C_{13}^{1} = 26624$,

The number of twelve-dimensional faces (simplexes) is $f_{12}(13) = 2^{13} = 4096.$

The obtained numbers determine the structure of the polytope of hereditary information.

The law of conservation of incidents in polytope of hereditary information

The monograph introduced the concept of the incidence coefficients of elements of lower dimension with respect to elements of the higher dimension and elements of higher dimension with respect to elements of the lower dimension. The first characterizes the number of elements of a certain higher dimension to which the given element of a lower dimension belongs. The second characterizes the number of elements of a given lower dimension that are included in a particular element of a higher dimension. Here we must remember that the vertices of geometric elements of various dimensions are atoms, molecules or functional groups. Therefore, the incidence of geometric elements to a friend means contact between particles of the matter, including living matter. The contact between particles of matter can be interpreted as the transfer of information on material structures, including biological structures. We introduce the notation: $k_{d_j^i d_u}$ is the number of elements of dimension u, which include an element of dimension j (u>j) with number i. Thus, $k_{d_j^i d_u}$ is the incidence factor of element i with dimension j relative to elements with dimension u. We introduce the notation also: $k_{d_j d_u^i}$ is the number of elements of dimension j, which included in element i with a dimension u (u > j). Thus, $k_{d_j d_u^i}$ is the incidence factor of element i with dimension u relatively to elements with dimension j [5].

The smallest dimension of the cross-polytope is 4. From (2) it follows that in this polytope there are 8 vertices, 24 edges, 32 two-dimensional faces, 16 three-dimensional faces (Figure 6). The factors of incidents (from smaller dimension to larger) are

$$k_{d_0^i d_1} = 6, i = 1 \div 8; k_{d_0^i d_2} = 12, i = 1 \div 8; k_{d_0^i d_3} = 8, i = 1 \div 8; k_{d_0^i d_4} = 1, i = 1 \div 8; k_{d_1^i d_2} = 4, i = 1 \div 24;$$
$$k_{d_1^i d_3} = 4, i = 1 \div 24; k_{d_1^i d_4} = 1, i = 1 \div 24; k_{d_2^i d_3} = 2, i = 1 \div 32; k_{d_2^i d_4} = 1, i = 1 \div 32; k_{d_3^i d_4} = 1, i = 1 \div 16.$$

Sum up the incidence coefficients for all vertices, edges, two-dimension faces and three-dimension faces of the 4-cross polytope

$$\sum_{i=1}^{8} k_{d_0^i d_1} + \sum_{i=1}^{8} k_{d_0^i d_2} + \sum_{i=1}^{8} k_{d_0^i d_3} + \sum_{i=1}^{8} k_{d_0^i d_4} + \sum_{i=1}^{24} k_{d_1^i d_2} + \sum_{i=1}^{24} k_{d_1^i d_3} + \sum_{i=1}^{24} k_{d_1^i d_4} +$$
$$\sum_{i=1}^{32} k_{d_2^i d_3} + \sum_{i=1}^{16} k_{d_3^i d_4} + \sum_{i=1}^{32} k_{d_2^i d_4} = 544. \quad (3)$$

The factors of incidents (from larger dimension to smaller, Figure 6) are

$$k_{d_0 d_1^i} = 2, i = 1 \div 24; k_{d_0 d_2^i} = 3, i = 1 \div 32; k_{d_0 d_3^i} = 4, i = 1 \div 16; k_{d_0 d_4} = 8; k_{d_1 d_2^i} = 3, i = 1 \div 32;$$
$$k_{d_1 d_3^i} = 6, i = 1 \div 16; k_{d_1 d_4} = 24; k_{d_2 d_3^i} = 4, \mathrm{i} = 1 \div 8; k_{d_2 d_4} = 32; k_{d_3 d_4} = 16.$$

Sum up the incidence coefficients for all elements of the 4-cross-polytope with dimension larger of zero

$$\sum_{i=1}^{24} k_{d_0 d_1^i} + \sum_{i=1}^{32} k_{d_0 d_2^i} + \sum_{i=1}^{32} k_{d_1 d_2^i} + \sum_{i=1}^{16} k_{d_0 d_3^i} + \sum_{i=1}^{16} k_{d_1 d_3^i} + \sum_{i=1}^{16} k_{d_2 d_3^i} + k_{d_0 d_4} + k_{d_1 d_4} + k_{d_2 d_4} + k_{d_3 d_4} = 544. \quad (4)$$

Comparing (3) and (4) you can see that the sum of incidents in a 4-cross-polytope from elements with a lower dimension to elements with a higher dimension is equal to the sum of incidents from elements with a higher dimension to elements with a lower dimension. Thus, the sum of incidents retains its value when changes the direction of the relationship between the elements (the law of conservation of incidents).

Theorem 1: In any cross-polytope of dimension n, the sum of all incidents of elements of a lower dimension with respect to all elements of a higher dimension is equal to the sum of all incidents of elements of a higher dimension with respect to all elements of a lower dimension and equals the sum of the series

$$f_0(n)3^{n-1} + f_1(n)3^{n-2} + f_2(n)3^{n-3} + ... + f_{n-1}(n)3^0, \ f_d(n) = C_n^{n-1-d} 2^{1+d}, d = 0 \div (n-1).$$

Proof According to equation (2), each cross-polytope of dimension n has 2n=vertices. The peculiarity of a cross-polytope is that each of its vertices has an opposite vertex, with which it is not connected by an edge. Moreover, there is one edge between this vertex and all other vertices. We subtract from the total number of vertices two vertices (the selected vertex and its opposite) 2n-2. This is the possible number of edges emanating from the selected vertex. Thus, the incidence coefficient of the edges of any vertex is

$$k_{d_0^i d_1} = 2(n-1) = 2\mathrm{C}_{n-1}^{1}, i = 1 \div f_0(n).$$

A cross-polytope of any dimension can be depicted as a projection on a two-dimensional plane. In this image, all its vertices are located on a circle, with the selected vertex and its opposite vertex located symmetrically relative to the center of the circle. A mentally drawn line through these two vertices halves the circle [5].

Therefore, the number of variants the location of the edges from the selected vertex to the other vertices in one of the halves of the circle is n-1, i.e. the number of combinations of n-1 vertices one by one. Since there are two halves of a circle, then for the total number of vertex selection options for edge formation, this number of combinations should be multiplied by 2. This is the meaning of the expression for the incidence rate of any vertex in the n-cross-polytope with respect to the edge. To further prove the theorem 1 and clarify the nature of the formation of the coefficients of incidence in the n-cross-polytope, we will use this technique. We arrange the vertices of the 4-cross-polytope in two lines, so that the vertices unlinked by an edge form vertical pairs: each vertex in the top line is not connected by an edge to the vertex in the bottom line located strictly under this vertex in the top line.

$$\begin{matrix} d_o^1 & d_o^2 & d_o^3 & d_o^4 \\ d_o^5 & d_o^6 & d_o^7 & d_o^8 \end{matrix}$$

We choose some vertex in the top line (5), for example d_o^1. It cannot be connected by an edge with a vertex d_o^5 in the bottom line, but it can be connected by an edge with other vertices of the bottom line. The number of such options is 3, i.e. with n=4 are $3 = C_{n-1}^1 = C_3^1$. However, a vertex d_o^1 may be connected by an edge with any of the remaining three vertices in the top line (5). Therefore, the incidence coefficient of a vertex with respect to an edge in 4-cross-polytope $k_{d_0^i d_1} = 2\mathrm{C}_{n-1}^1 = 6, i = 1 \div 8.$ when considering the belonging of a vertex d_o^1 to two-dimensional faces, it is necessary to determine the number of options for the participation of two vertices in the top line (5) (without a vertex d_o^1) and in the bottom line (5) (without a vertex d_o^5). This will be the number $2C_3^1$. In addition, as the vertices of the triangle, there may be vertices located in the upper and lower lines (5) in a cross-section way.

There are 6 such variants in (5), i.e. more $2C_3^1$. Therefore, the incidence coefficient of a vertex with respect to a two-dimensional element in 4-cross-polytope is $k_{d_0^i d_2} = 2^2\,\mathrm{C}_{n-1}^2 = 12, i = 1 \div 8.$ multiplying this number by the number of vertices in 4-cross-polytope you can get the total number of incidents of vertices to two- dimensional elements in 4-cross-polytope equal to 96. This number coincides with the corresponding number, defined earlier. Combination d_o^1 with three the different vertices from (5) can get the incidence coefficient of vertex to the three-dimension body (tetrahedron) for condition absent in combination opposite vertices. Can to show that this combination is 8 $k_{d_0^i d_3} = 2^3 C_3^3 = 8, i = 1 \div 8.$; multiplying this number by the number of vertices in 4-cross-polytope you can get the total number of incidents of vertices to three-dimensional elements in 4-cross-polytope equal to 64. This number coincides with the corresponding number, defined earlier. Combination two vertices, for example d_o^1 and d_o^2 (the edge d_o^1 d_o^2) with two the different vertices from (5) you can get the incidence coefficient of edge to three-dimension body (tetrahedron) for condition absent in combination opposite vertices. Can to show that this combination is 4 $k_{d_1^i d_3} = 2^2 C_2^2 = 4, i = 1 \div 8.$; Multiplying this number by the number of edges in 4-cross-polytope you can get the total number of incidents of edges to three- dimensional elements in 4-cross-polytope equal to 96. This number coincides with the corresponding number, defined earlier.

Combination two vertices (the edge d_o^1 d_o^2) with one the vertex from (5) can get the incidence coefficient of edge to triangle for condition absent in combination opposite vertices. Can to show that this combination is 4 $k_{d_1^i d_2} = 2C_2^1 = 4, i = 1 \div 8.$, multiplying this number by the number of edges in 4-cross- polytope you can get the total number of incidents of edges to triangle in 4-cross-polytope equal to 24. This number coincides with the corresponding number, defined earlier. In general case can write of 2n vertices of the n-cross-polytope in two lines

$$\begin{matrix} d_o^1 & d_o^2 & d_o^3 & \ldots & d_o^n \\ d_o^5 & d_o^6 & d_o^7 & \ldots & d_o^{2n} \end{matrix}, \qquad (6)$$

Where vertex in the top line is not connected by an edge to the vertex in the bottom line located strictly under this vertex in the top line. The incidence coefficient of a vertex with respect to an edge in n-cross-polytope is $k_{d_0^i d_1} = 2\mathrm{C}_{n-1}^1, i = 1 \div f_0(n).$
The incidence coefficient of a vertex with respect to a two-dimensional element in n-cross-polytope is $k_{d_0^i d_2} = 2^2\,\mathrm{C}_{n-1}^2, i = 1 \div f_0(n).$ the incidence coefficient of a vertex with respect to a three-dimensional edge in n-cross-polytope is $k_{d_0^i d_3} = 2^3\,\mathrm{C}_{n-1}^3, i = 1 \div f_0(n).$ Go on can to say the incidence coefficient of a vertex with respect to a (n-1)-dimensional element in n-cross-polytope $k_{d_0^i d_{n-1}} = 2^{n-1}\,\mathrm{C}_{n-1}^{n-1}, i = 1 \div f_0(n).$ Obviously, that the incidence coefficient of a vertex with respect to n-cross-polytope is $k_{d_0^i d_n} = 1, i = 1 \div f_0(n).$ Multiplying of the incidence coefficients of a vertex with respect to different dimension elements in n-cross-polytope and sum the product you can get the common express for the number of incident vertices to elements of different dimension in a n-cross-polytope

$$f_0(2^0 + 2^1 C_{n-1}^1 + 2^2 C_{n-1}^2 + \ldots + 2^{n-1} C_{n-1}^{n-1}) = f_0(n)\sum_{i=0}^{n-1} 2^i C_{n-1}^i = f_0(n)3^{n-1}.$$

In this way you can get the common express for the number of incident edges to elements of different dimension more one in n-cross-polytope,

$$f_1(2^0 + 2^1 C_{n-2}^1 + 2^2 C_{n-2}^2 + \ldots + 2^{n-2} C_{n-2}^{n-2}) = f_1(n)\sum_{i=0}^{n-2} 2^i C_{n-2}^i = f_1(n)3^{n-2}$$

And go on. In result, one can get the common express for the sum of all incidents of elements of a lower dimension with respect to all elements of a higher dimension in a n-cross-polytope

$$f_0(n)3^{n-1} + f_1(n)3^{n-2} + \ldots + f_{n-1}3^0 = \sum_{i=0}^{n-1} f_i(n)3^{n-1-i},\ f_i(n) = C_n^{n-1-i}2^{1+i}. \qquad (7).$$

Let $k_{d_h d_j^i}$ be the number of elements of dimension h in an n-cross-polytope belonging to some one element of dimension j $(h < j)$ that is d_j^i. Obviously, this number is equal to $f_h(j)_s$ for simplex, and $i = 1 \div f_j(n)_{cr}$ for n-cross-polytope. So, elements of a cross-polytope are simplexes the product $f_h(j)_s\, f_j(n)_{cr}$ corresponds to the number of elements of dimension h belonging to all elements of dimension j for a simplex. This product is equal to $f_d(n) = C_{n+1}^{d+1}$

$$C_{j+1}^{h+1} 2^{1+j} C_n^{n-1-j} = 2^{1+j} \frac{(j+1)!}{(h+1)!(j-h)!} \frac{n!}{(n-1-j)!(1+j)!}.$$

Let us compare this number with the number of elements of dimension j, which have elements of dimension h in the n-cross-polytope.

$$k_{d_h^i d_j} f_h(n)_{cr} = C_n^{n-1-h} 2^{1+h} C_{n-h-1}^{j-h} 2^{j-h} = 2^{1+j} \frac{n!}{(n-1-h)!(1+h)!} \frac{(n-h-1)!}{(n-j-1)!(j-h)!}$$

Obviously, these numbers are equal to each other. This proves that the number of elements of dimension h in an n-cross-polytope belonging to all dimensions of j $(j > h)$ in an n-cross-polytope is equal to the number of elements of dimension j, which have elements of dimension h as elements.

Thus, the total number of incidents of elements of a smaller dimension with respect to elements of a higher dimension is equal to the total number of incidents of elements of a higher dimension with respect to elements of a smaller dimension. The total number defines expression (7). Q.E.D The polytope of hereditary information is a cross-polytope of dimension 13. Substituting the value n=13 and the values of $f_i(n)$ calculated in the previous section in expression (7), we find the total value of the incident flow in the polytope of hereditary information.

$$\sum_{i=0}^{12} f_i(13) 3^{12-i} = 1.78 \cdot 10^9$$

A significant value of the total incidence stream in the polytope of hereditary information indicates an intensive exchange of information between elements of the polytope of hereditary information. For example, this value is 400 times larger than the incident flux in a simplex of dimension 13. This may explain for the recently discovered epigenetic principle of the transmission of hereditary information without changing the sequence of genes in DNA and RNA molecules [8].

Conclusion

The representations of the sugar molecule and the residue of phosphoric acid in the form of polytopes of higher dimension are used. Based on these ideas and their simplified three-dimensional images, a three-dimensional image of nucleic acids is constructed. The geometry of the neighborhood of the compound of two nucleic acid helices with nitrogen bases has been investigated in detail. It is proved that this neighborhood is a polytope with anti-parallel edges of dimension 13 (13-cross-polytope). This polytope is called of the polytope of hereditary information. The geometry of the polytope of hereditary information is investigated. It is shown that in the flat coordinate planes of the polytope of hereditary information there are located flat complementary hydrogen compounds of the nitrogenous bases of two nucleic acid helices. It turned out that the number of these coordinate planes (12) is exactly as many as there are various options for hydrogen compounds of nitrogenous bases. Thus, in each of these coordinate planes one of the possible types of bonding of nitrogenous bases is located.

Thus, the possible orientation of flat nitrogenous bases in the space of higher dimension is determined. An image of the polytope of hereditary information with a specific indication of its coordinate planes is constructed. The incidence of low-dimensional elements to the highest-dimensional elements of this polytope is studied, as well as the incidence of higher-dimensional elements to low-dimensional elements of this polytope. The values of the total incidence flows from low-dimensional elements to higher-dimensional elements and vice versa are determined. These values turned out to be equal (the law of conservation of incidents) and exceeding one billion. This indicates an intensive flow of information between the elements of the polytopic of hereditary information, ensuring the transmission of hereditary information. This can serve, in particular, to explain the existence of the transmission of hereditary changes without changing the sequence of genes (epigenetics).

References

1. Watson JD, Crick FHC. Molecular structure of nucleic acids (1953a) Nature 171: 738-740. https://doi.org/10.1038/171737a0
2. Zhizhin GV. The structure, topological and functional dimension of biomolecules (2016) J Chemoinform Chemical Eng 5: 44-58. https://doi.org/10.4018/IJCCE.2016070104
3. Zhizhin GV. Chemical Compound Structures and the Higher Dimension of Molecules: Emerging Research and Opportunities (2018a) IGI Global. https://doi.org/10.4018/978-1-5225-4108-0
4. Zhizhin GV. The metric structure of nucleic acids and higher dimension of their constituents (2018b) J Chemoinform Chemical Eng 7: 1-15. https://doi.org/10.4018/IJCCE.2018070101
5. Zhizhin GV. To the question of the geometry of the structure of nucleic acids (2019a) J European Soci Math Chem 1: 1-11.
6. Zhizhin GV. The geometry of higher-dimensional polytopes (2019b) IGI Global.
7. Zhizhin GV. Attractors and higher dimensions in population and molecular biology: emerging research and opportunities (2019c) IGI Global. http://doi.org/10.4018/978-1-5225-6968-8
8. Zhizhin GV. The law of conservation of incidents in the space of nano world (2019d) J. Chemoinformatics and Chem Eng. http://dx.doi.org/10.4018/IJCCE.2019010103
9. Spirin AS. The ribosomes and protein biosynthesis (2019) Mol biology.
10. Lindquist S, Chakrabortee S, Kayatekin C, Newby GA, Mendillo ML et al. Luminidependens (LD) is an Arabidopsis protein with prion behavior (2016) Proceedings National Sci 113: 6065-6070. https://doi.org/10.1073/pnas.1604478113
11. Mancuso S. Plant Revolution. Plants have already invented our future (2017) Guitue Publishers, Italy.
12. Sanbonmatsu KY, Hawkes EJ, Hennelly SP, Novikova IV, Irwin JA, et al. Coolair antisense RNAs from evolutionarily conserved elaborate secondary structures (2016) Cell reports 16: 3087-3096. https://doi.org/10.1016/j.celrep.2016.08.045
13. Lehninger AL. Principles of Biochemistry (1982) Worth Publishers, United States.
14. Poincare A. Analysis situs (1895) J Polytechnic School 1: 1-121.
15. Watson JD, Crick FHC. Genetical implications of the structure of deoxyribose nucleic acid (1953b) Nature 171: 964-967. https://doi.org/10.1038/171964b0
16. Spirin AS, Gavrilova LP. The Ribosome (1971) Science, Moscow, Russia.
17. Frank KM. DNA structure: A simple solution to the stability of the double helix? (1986) Nature 324: 305. https://doi.org/10.1038/324305a0
18. Frank KM. H-form DNA and the hairpin-triplex model (1988) Nature 333: 214. https://doi.org/10.1038/333214b0
19. Grunbaum B. Günter M. Ziegler. Convex Polytopes (1967) Springer, London, New York. https://doi.org/10.1007/978-1-4613-0019-9
20. Mancuso S and Viola A. Sensitivity and intelligence of the plant world (2013) Guitue Publishers, Italy.